

Uniqueness in Logic Puzzles

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Pure deduction puzzles typically have a single unique solution. However, some puzzle setters argue that challenges with multiple solutions are also valid, if they can be solved by eliminating choices that lead to ambiguous states. This paper considers the arguments for and against this position, and presents a counterexample that demonstrates the danger of using uniqueness to decide between multiple solutions.

1 INTRODUCTION

A CHARACTERISTIC of pure deduction puzzles, such as Japanese logic puzzles, is that each challenge has a single unique solution. This allows such challenges to be solved by deduction rather than guesswork [1].

I was therefore surprised to find a Kakuro challenge with multiple solutions in a publication as respectable as *The Guardian* [2]. This is the first time that I had ever encountered such a case in print. The aim in Kakuro is to fill each cell with a digit in the range 1–9, such that each horizontal and vertical run adds to the hint total shown, and no digit is repeated within each run [3].

Figure 1 shows the relevant section of the Kakuro challenge in question (all other values have been resolved). Possible values for the final few unresolved cells are shown in small print, and a key cell with possible values 4 or 5 circled. This challenge has three possible solutions, depending on whether this key cell takes the value 4 or 5, as shown.

After alerting the UK setter of this challenge to what appeared to be a flawed design with no deducible solution, I was also surprised by his response. He maintained that this challenge was indeed valid, and could be solved by deduction based on *relative* uniqueness.

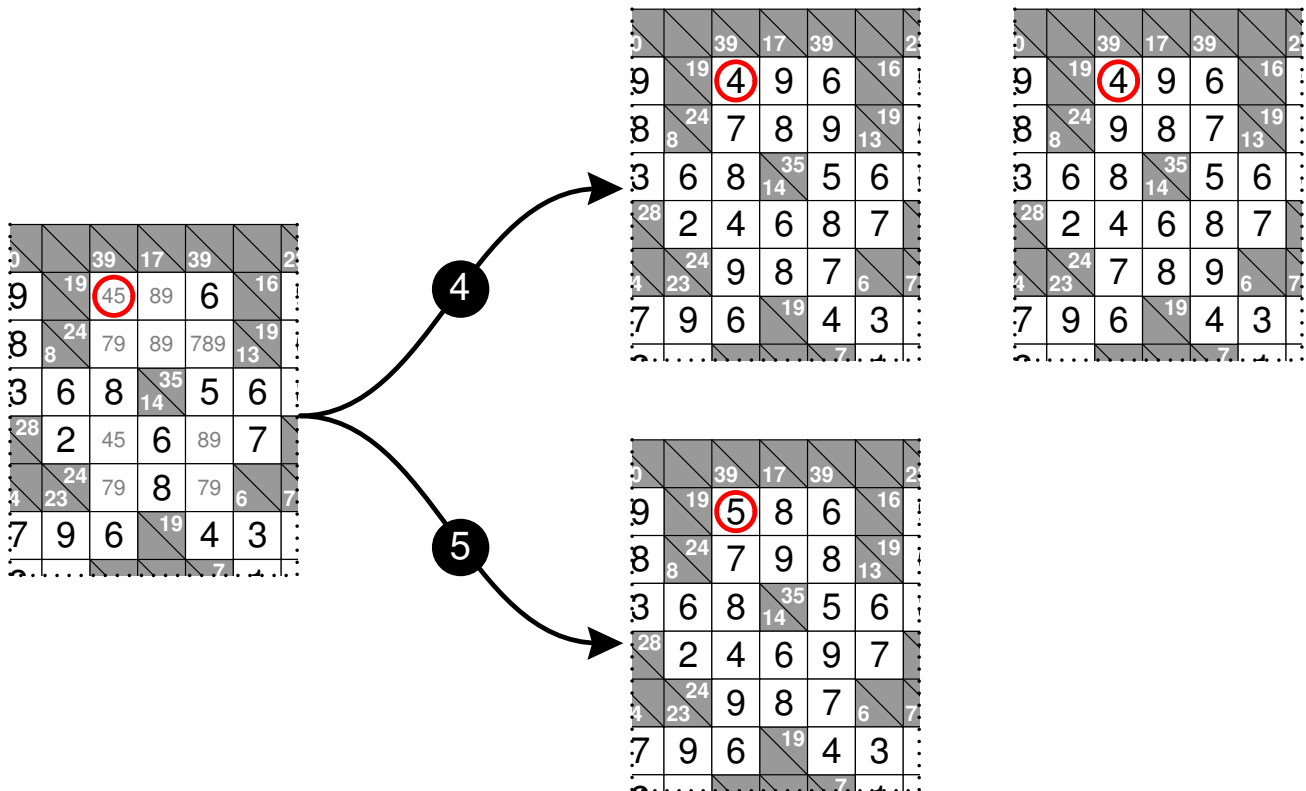


Figure 1. A Kakuro challenge with three solutions. The circled cell can take the value 4 or 5.

(left), hence this move must be an edge and c must be the ‘correct’ solution (right).

However, if the same process is applied to the move indicated in Figure 4 (top, dotted), then b is deduced to be the ‘correct’ solution (right).

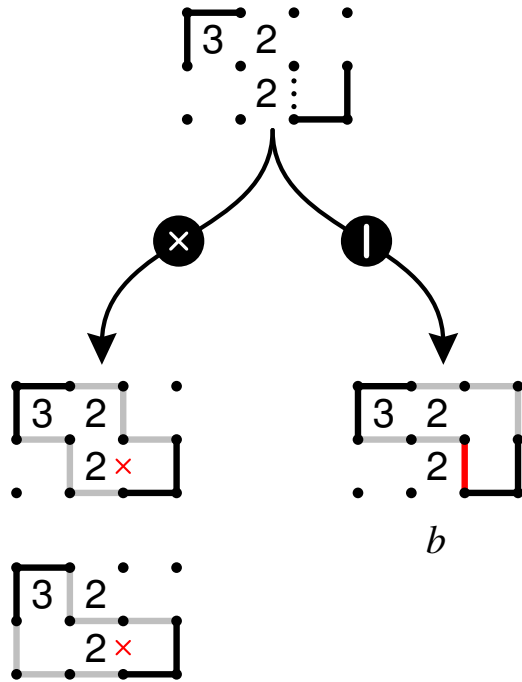


Figure 4. Deduction by uniqueness yields b .

Deduction by relative uniqueness therefore gives two conflicting ‘correct’ solutions, b and c , depending on processing order. To derive the same solution as the setter, the solver would have to follow the same sequence of decisions in the exact same order, but there is no way to enforce this in practice. Deduction by relative uniqueness is not guaranteed to yield the same solution from among multiple solutions in all cases.

This Slitherlink counterexample could be said to have one valid solution (depending on the order in which the solver made their deductions), two equally valid solutions (through deduction by relative uniqueness) or three equally valid solutions (which it does, after all – see Figure 2). This is clearly an unsatisfactory state of affairs. But if absolute uniqueness is enforced, and such cases of multiple solutions avoided, then all of these problems simply go away, at no real cost. As expert puzzle designer Hiroshi Higashida points out: *puzzle creators, not only solvers, mustn’t defy rules*,

either [5, p216].

4 CONCLUSION

The characteristic of pure deduction puzzles to have a single unique solution is not only elegant, but performs a vital practical function. It guarantees that challenges can be solved by deduction alone, without guesswork or ambiguity, and means that the setter and solver are both playing from the same rule set without the need to make assumptions about implied or hidden rules. Further, uniqueness makes challenges self-checking; if the player has deduced a solution, then it must be the correct one. As tempting as it may be to relax this constraint of absolute uniqueness and instead exploit relative uniqueness as a solution strategy, this is best avoided in pure deduction puzzles.

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